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MA 16020 Exam 3 Memo
Monday, September 11, 2023
6:30-7:30pm (plan to arrive no later than 6:15pm)

1. The exam will consist of 12 multiple choice questions covering lessons 1-7.
2. The exam location will depend on your instructor. The exams will be given at the following locations:
   (a) ARMS 1010: Ben Doyle
   (b) FRNY G140: Jakayla Robbins
   (c) RHPH 172: Victor Hughes
   (d) PHYS 114: Alexandra Cuadra/Dave Norris
3. You will be emailed an assigned seat before the exam. Bring this seating assignment with you to the exam.
4. Only a TI-30Xa calculator will be allowed on the exam. No other calculator will be allowed.
5. You MUST bring your PUID to the exam.
6. Since the exams will be machine graded, the only thing that will be graded is the scantron sheet. Make sure that you have correctly filled in all of the information (name, PUID, test form number, section number, and all of your answers) on the answer sheet.

   200
7. Please reread the section on the syllabus regarding exams. Exceptions for any make-up exams are listed on the syllabus
8. There are review problems in LON-CAPA under Contents->Exam Review->Exam 1 Review. These review problems allow three entries before it will show the answer. You can get additional randomizations of each problem. There are also old exams for practice in LON-CAPA. The questions that are relevant for this exam are:

   Exam 1 Fall 2022: #1, 2, 3, 4, 5, 6, 7

   Exam 1 Spring 2023: #1, 2, 3, 4, 5, 6
9. No one will be permitted to leave during the first 20 minutes of the exam; after the first 20 minutes, no one will be permitted to take exam.

Scratch paper? - Answer on Wednesday

Lesson 6 Integration with Natural Logs

I. Review of ln function

II. Examples

I. Review of ln function

(A) ln Rules $a, b > 0$, x is a real #

(1) $\ln(ab) = \ln(a) + \ln(b)$

(2) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

(3) $\ln(e^x) = x$ (Special case of (7) & (5))

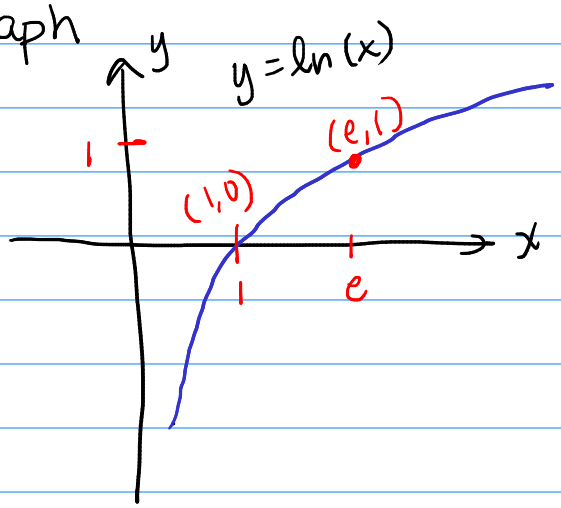
(4) $e^{\ln(x)} = x$

(5) $\ln(e) = 1$

(6) $\ln(1) = 0$

(7) $\ln(a^x) = x\ln(a)$

(B) Graph



domain: $x > 0$

$(0, \infty)$

range: All real #'s

$\mathbb{R}$

$(-\infty, \infty)$

(C) Calculus

$$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x} = x^{-1}$$

$$\frac{d}{dx}\left[\ln(|x|)\right] = \frac{1}{x}$$

New

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln(|x|) + C$$

II. Examples

Strategy 1 If $\int$ contains an $\ln(\ )$ and a $\frac{1}{x}$, try $u = \ln(\ )$

Ex $\int_{\frac{e^{10}}{3}}^{\frac{e^{20}}{3}} \frac{5}{x\ln(3x)}\,dx$

$u = \ln(3x)$

$du = \frac{1}{3x}\cdot 3\,dx = \frac{1}{x}dx$

$x = \frac{e^{10}}{3} \Rightarrow u = \ln\left(3\cdot\frac{e^{10}}{3}\right) = \ln(e^{10}) = 10$

$x = \frac{e^{20}}{3} \Rightarrow u = \ln\left(3\cdot\frac{e^{20}}{3}\right) = \ln(e^{20}) = 20$

$= \int_{10}^{20} \frac{5}{u}\,du = 5\ln(|u|)\Big|_{10}^{20}$

$= 5\ln(20) - 5\ln(10)$

Strategy 2 If $\int \frac{\triangle}{\square}\,dx$

Try $u = \square$

Does $\triangle\,dx$ contain du (up to a constant multiple)?

Ex $\int \frac{5x}{x^2+1}\,dx$ ($5x\,dx = \frac{1}{2}du$, $x^2+1 = u$)

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int \frac{5}{u}\cdot\frac{1}{2}du = \frac{5}{2}\int \frac{1}{u}\,du$

$= \frac{5}{2}\ln(|u|) + C$

$= \frac{5}{2}\ln(|x^2+1|) + C$

$= \frac{5}{2}\ln(x^2+1) + C$

EX $\int \tan(3x)\,dx = \int \frac{\sin(3x)}{\cos(3x)}\,dx$ ($-\frac{1}{3}du$; u) $= \int \frac{1}{u}\cdot\frac{-1}{3}\,du$

$u = \cos(3x)$

$du = -\sin(3x)\cdot 3\,dx$

$du = -3\sin(3x)\,dx$

$-\frac{1}{3}du = \sin(3x)\,dx$

$= \frac{-1}{3}\int \frac{1}{u}\,du$

$= \frac{-1}{3}\ln(|u|) + C$

$= \frac{-1}{3}\ln(|\cos(3x)|) + C$

$= \ln\left(|\cos(3x)|^{-1/3}\right) + C$

$= \ln\left(\frac{1}{|\cos(3x)|^{1/3}}\right) + C$

$= \ln\left(|\sec(3x)|^{1/3}\right) + C$

$= \ln\left(\sqrt[3]{|\sec(3x)|}\right) + C$

EX Find the average value of $f(x) = \frac{2}{3x\ln(x^{10})}$ on the interval $e^3 \le x \le e^8$

$$f_{ave} = \frac{\int_{e^3}^{e^8} \frac{2}{3x\ln(x^{10})}\,dx}{e^8 - e^3} = \frac{1}{e^8 - e^3}\int_{e^3}^{e^8} \frac{2}{3x\ln(x^{10})}\,dx$$

($\frac{1}{10}du$; u)

$u = \ln(x^{10})$

$du = \frac{1}{x^{10}}\cdot 10x^9\,dx$

$du = \frac{10}{x}\,dx$

$\frac{1}{10}du = \frac{1}{x}\,dx$

$x = e^3 \Rightarrow u = \ln((e^3)^{10}) = \ln(e^{30}) = 30$

$x = e^8 \Rightarrow u = \ln((e^8)^{10}) = \ln(e^{80}) = 80$

$$\frac{1}{e^8 - e^3}\int_{30}^{80} \frac{2}{3}\cdot\frac{1}{u}\cdot\frac{1}{10}\,du = \frac{2}{30(e^8 - e^3)}\cdot\ln\left(\frac{8}{3}\right)$$